QUANTUM GRAVITY, HIGHER DERIVATIVES & NONLOCALITY

東京工業大学 Tokyo Institute of Technology

UNIVERSITÀ DEGLI STUDI DI SALERNO Dipartimento di Fisica E.R. Caianiello

INFN Istituto Nazionale di Fisica Nucleare

Lorentz non-invariance as the palladium of locality, unitarity and renormalizability in quantum gravity

A.O.Barvinsky

Theory Department, Lebedev Physics Institute, Moscow

with D. Blas
M. Herrero-Valea
A.Kamenshchik
N. Kolganov
A. Kurov
D. Nesterov
S. Sibiryakov
C. Steinwachs
A.Vikman

Plan

Introduction: The tale of two models

I. Horava gravity

1) Renormalizability and regular gauges: projectable models
2) BRST structure of renormalization and background formalism
3) Asymptotic freedom in (2+1)-dimensional model (towards (3+1)-dimensional case)

II. Generalized unimodular gravity (GUMG)

1) Motivation for UMG --> GUMG
2) Unitarity, inflation, naturalness
3) Covariantization and k-essence, cuscutons and gradient expansion
4) Renormalizability (?), generalized GUMG + generalized RG (??))

D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B.,
PRD 93, 064022 (2016), arXiv:1512.02250; arXiv:1705.03480;
PRL 119, 211301 (2017), arXiv:1706.06809;
M. Herrero-Valea, S. Sibiryakov & A.B., PRD100 (2019) 026012;
N. Kolganov, A. Kurov, D. Nesterov & A.B, PRD100 (2019), 023542;
N. Kolganov & A.B., arXiv:1908.05697, *N. Kolganov , A.Vikman & A.B.,*
arXiv:2011.0651

Renormalization of Horava gravity

Saving unitarity in renormalizable QG

Einstein GR $$S_{EH} = \frac{M_P^2}{2} \int dtd^dx\ R$$

nonrenormalizable

$$\Rightarrow \frac{M_P^2}{2} \int dtd^dx \left(h_{ij}\Box h_{ij} + h^2\Box h + \dots\right)$$

Higher derivative gravity

Stelle (1977)

$$\int \left(M_P^2 R + R_{\mu\nu}R^{\mu\nu} + R^2\right)$$

$$\Rightarrow \int \left(M_P^2 h_{ij}\Box h_{ij} + h_{ij}\Box^2 h_{ij} + \dots\right)$$

dominates at $k \gg M_P$

The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)
Avramidy & A.B. (1985)

But has ghost poles $\Rightarrow$ no unitary interpretation

Horava (2009)

$$\underbrace{\int dt\, d^d x}_{\propto\, b^{-(z+d)}} \left(\dot{h}_{ij}\dot{h}_{ij} - h_{ij}(-\Delta)^z h_{ij} + \dots\right)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\,, \qquad t \mapsto b^{-z} t \quad \Longrightarrow \quad h_{ij} \mapsto b^{(d-z)/2} h_{ij}$$

Critical theory in $z = d$

LI is necessarily broken. We want to preserve as many symmetries, as possible

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \quad \Longrightarrow \quad \gamma_{ij} \quad N^i, \qquad i = 1, \dots, d$$

$$t \mapsto \tilde{t}(t) \quad \Longrightarrow \quad N$$

Foliation preserving diffeomorphisms

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \ , \quad t \mapsto \tilde{t}(t)$$

ADM metric decomposition

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ , \quad i, j = 1, \ldots, d$$

space dimensionality

Anisotropic scaling transformations and scaling dimensions

$$x^i \to \lambda^{-1} x^i, \quad t \to \lambda^{-z} t, \quad N^i \to \lambda^{z-1} N^i, \quad \gamma_{ij} \to \gamma_{ij},$$

$$[x] = -1, \ [t] = -z, \ [N^i] = z - 1, \ [\gamma_{ij}] = 0, \qquad [K_{ij}] = z.$$

extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

``Projectable" theory $N = \text{const} = 1$

kinetic term -- unitarity

Horava gravity action

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma} N \Big(\underbrace{K_{ij}K^{ij} - \lambda K^2}_{} - \mathcal{V}(\gamma) \Big)$$

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Potential term

$$\mathcal{V}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R^{ij} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$
$$+ \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \ldots$$

Many more versions: extra structures in non-projectable theory, reduction of structures for detailed balance case . . .

$$N \neq \text{const}, \quad a_i = \nabla_i \ln N, \ldots$$

$d + 1 = 4$ **DoF: tt-graviton and scalar**

Unitarity domain (no ghosts) $\frac{1-\lambda}{1-3\lambda} > 0$

$$\omega_{tt}^2 = \eta k^2 + \mu_2 k^4 + \nu_5 k^6 \ ,$$
$$\omega_s^2 = \frac{1-\lambda}{1-3\lambda}\Big(- \eta k^2 + (8\mu_1 + 3\mu_2)k^4 + (8\nu_4 + 3\nu_5)k^6\Big)$$

↑

tachyon in IR

Divergences power counting

$$\text{Deg of div} \int \frac{d^{d+1}p}{\left(p^2\right)^N} = d + 1 - 2N = \text{physical dimensionality}$$

$$\Downarrow \qquad p = (\omega, \mathbf{k}), \quad p^2 \to \omega^2 + \mathbf{k}^{2z}$$

$$\text{Deg of div} \int \frac{d\omega\, d^d k}{\left(\omega^2 + \mathbf{k}^{2z}\right)^N} = z + d - 2zN = \text{scaling dimensionality}$$

physical dimensionality $\neq$ scaling dimensionality

$$z = d$$

critical value

$$\mathcal{V}^{(d=2)}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2$$

$$\mathcal{V}^{(d=3)}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}^2 + O(R^3, R\nabla^2 R)$$

Things are not so simple: power counting is not enough:

$$\int \prod_{l=1}^{L} d^{d+1}k^{(l)}\, \mathcal{F}_n(k) \prod_{m=1}^{M} \frac{1}{\left(P^{(m)}(k)\right)^2} \Longrightarrow \int \prod_{l=1}^{L} d\omega^{(l)} d^d \mathbf{k}^{(l)}\, \mathcal{F}_n(\omega, \mathbf{k}) \prod_{m=1}^{M} \frac{1}{A_m \left(\Omega^{(m)}(\omega)\right)^2 + B_m \left(\mathbf{K}^{(m)}(\mathbf{k})\right)^{2z}}$$

Generalization of BPHZ renormalization theory (subtraction of subdivergences) works only for $A_m > 0$ and $B_m > 0$

depends on gauge fixing

Invention of regular gauges for projectable HG

$$F^{\mu} \equiv \partial_{\nu} h^{\nu\mu} + \ldots \;\Rightarrow\; F^{i} = \dot{N}^{i} + c\partial_j \Delta h^{ji} + \ldots$$

$$[F^i] = 3 \;\Rightarrow\; [\mathcal{O}_{ij}] = -2, \quad \mathcal{O}_{ij} = (\Delta\delta_{ij} + \xi\partial_i\partial_j)^{-1}$$

extra derivatives to have homogeneity in scaling

Gauge fixing term

$$S_{\mathrm{gf}} = \frac{\sigma}{2G} \int dt\, d^d x\, \sqrt{\gamma}\, F_i\, \mathcal{O}^{ij} F_j$$

σ, ξ free gauge fixing parameters

⇩

Projectable HG is renormalizable in any d

Gauge invariance of counterterms

Background covariant gauge conditions + BRST structure of renormalization

Background field method:

$$\gamma_{ij} = \bar{\gamma}_{ij} + h_{ij}, \ \ \partial_i \to \bar{\nabla}_i, \ \ \mathcal{O}^{-1\,ij} = \bar{\Delta}\,\bar{\gamma}^{ij} + \xi\bar{\nabla}^i\bar{\nabla}^j, \ \ldots$$

DeWitt, Tyutin, Voronov, Stelle, Batalin, Vilkovisky, Slavnov, Arefieva, Abbott…
Barnich, Henneaux, Grassi, Anselmi,…

Blas, Herrero-Valea, Sibiryakov, Steinwachs & A.B.
arXiv: 1705.03480, JHEP07(2018)035

Background field extension of the BRST operator + inclusion of generating functional sources into the gauge fermion

⇩

1. BRST structure of renormalization via decoupling of the background field
2. No power counting or use of field dimensionalities
3. Extension to Lorentz symmetry violating theories
4. Extension to (nonrenormalizable) effective field theories

Extended BRS operator and gauge fermion $Q \to Q_{\mathrm{ext}}, \quad \Psi \to \Psi_{\mathrm{ext}}[\,\Phi, J\,]$

$$e^{-W/\hbar} = \int D\Phi\, e^{-(S+Q\Psi+J\Phi)/\hbar} \quad \Rightarrow \quad e^{-W/\hbar} = \int D\Phi\, e^{-(S+Q_{\mathrm{ext}}\Psi_{\mathrm{ext}})/\hbar}$$

Gauge independence on shell $\delta_\Psi W[\,J\,]\Big|_{J=0} = 0$

Renormalization:

$$S[\,\varphi\,] \to S[\,\varphi\,] + \Delta_\infty S[\,\varphi\,]$$

physical gauge invariant local counterterm

$$\boldsymbol{\Psi}_{\mathrm{ext}}[\,\Phi\,] \to \boldsymbol{\Psi}_{\mathrm{ext}}[\,\Phi\,] + \Delta_\infty \boldsymbol{\Psi}_{\mathrm{ext}}[\,\Phi\,]$$

local counterterm to gauge fermion ***(irrelevant)***

Asymptotic freedom in (2+1)-dimensions

$$S = \frac{1}{2G}\int dt\, d^2x\; N\sqrt{\gamma}\;\left(K_{ij}K^{ij} - \lambda K^2 + \mu R^2\right)$$

Off-shell extension is not unique:

$$\Gamma_{1-\mathrm{loop}} \to \Gamma_{1-\mathrm{loop}} + \int dt\, d^dx\, \Omega_{ij}\frac{\delta S}{\delta\gamma_{ij}}$$

Essential coupling constants:

$$\lambda, \quad \mathcal{G} \equiv \frac{G}{\sqrt{\mu}}$$

$$S_{\rm gf} = \frac{\sigma}{2G}\int dt\, d^2x\, \sqrt{\gamma}\, F_i\, \mathcal{O}^{ij} F_i$$

background covariant
gauge-fixing term
σ, ξ – free parameters

$$F_i = \partial_t n_i + \frac{1}{2\sigma}\mathcal{O}^{-1}_{ij}(\nabla^k h^j_k - \lambda \nabla^j h)$$

$$\mathcal{O}^{ij} = -[\gamma_{ij}\Delta + \xi \nabla_i \nabla_j]^{-1}$$

⇓

Mathematica package xAct

D. Brizuela, J. M. Martin-Garcia, and G. A. Mena Marugan, Gen. Rel. Grav. 41, 2415 (2009), arXiv:0807.0824

$$\beta_\lambda = \frac{15-14\lambda}{64\pi}\sqrt{\frac{1-2\lambda}{1-\lambda}}\,\mathcal{G}$$

$$\beta_{\mathcal{G}} = -\frac{(16-33\lambda+18\lambda^2)}{64\pi(1-\lambda)^2}\sqrt{\frac{1-\lambda}{1-2\lambda}}\,\mathcal{G}^2$$

Powerful check – gauge independence of essential couplings

Check in conformal gauge $h_{ij} = e^{2\phi}\gamma_{ij}$

$$\beta_\mu = \frac{2 - 7\lambda + 6\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda)}} G\sqrt{\mu}$$

$$\beta_G = -\frac{6\lambda - 7}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}} \frac{G^2}{\mu}$$

$\Rightarrow \beta_{\mathcal{G}}$

same

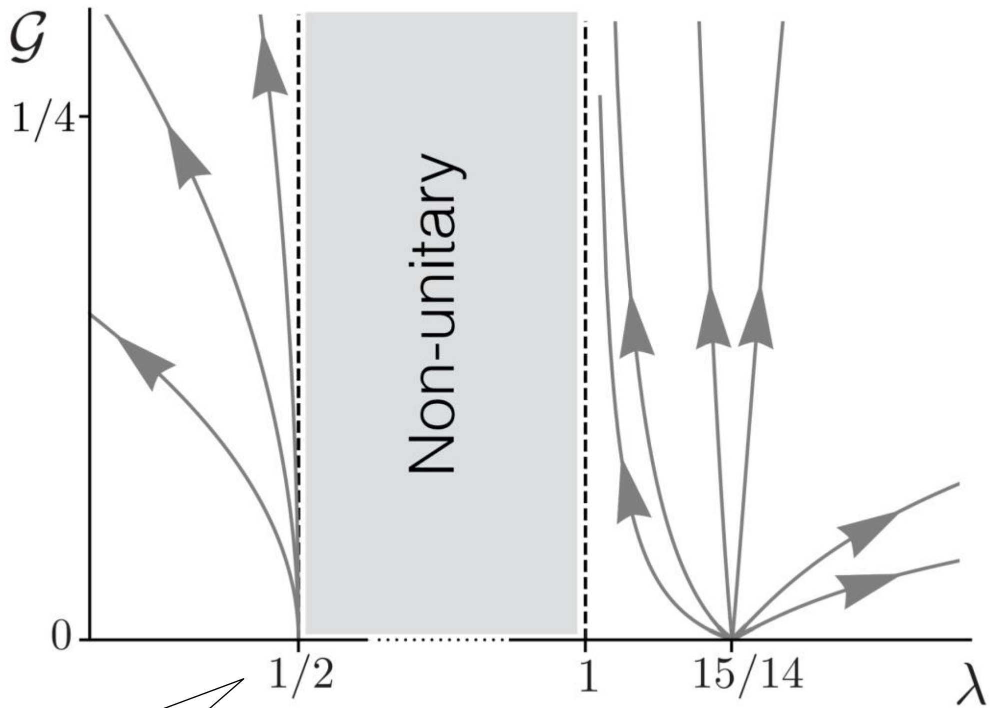
Compare to regular ``relativistic'' gauge

$$\beta_\mu = \frac{30 - 73\lambda + 42\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda)}} G\sqrt{\mu}$$

$$\beta_G = -\frac{30\lambda - 23}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}} \frac{G^2}{\mu}$$

$\Rightarrow \beta_{\mathcal{G}}$

Renormalization flows:



strongly coupled f. point

$$\mathcal{G} \to \tilde{\mathcal{G}} = \frac{G}{\sqrt{1-2\lambda}}$$

$$\beta_{\tilde{\mathcal{G}}} = -\frac{(1-2\lambda)^2}{64\pi(1-\lambda)^{3/2}}\,\tilde{\mathcal{G}}^2$$

AF UV fixed point

Cf. conformal truncation of (2+1)D HG, Benedetti and Guarneri, JHEP 03(2014)078: f. point λ=1/2, unitarity at G<0, λ>1

Towards RG flows of (3+1)-dimensional Horava gravity

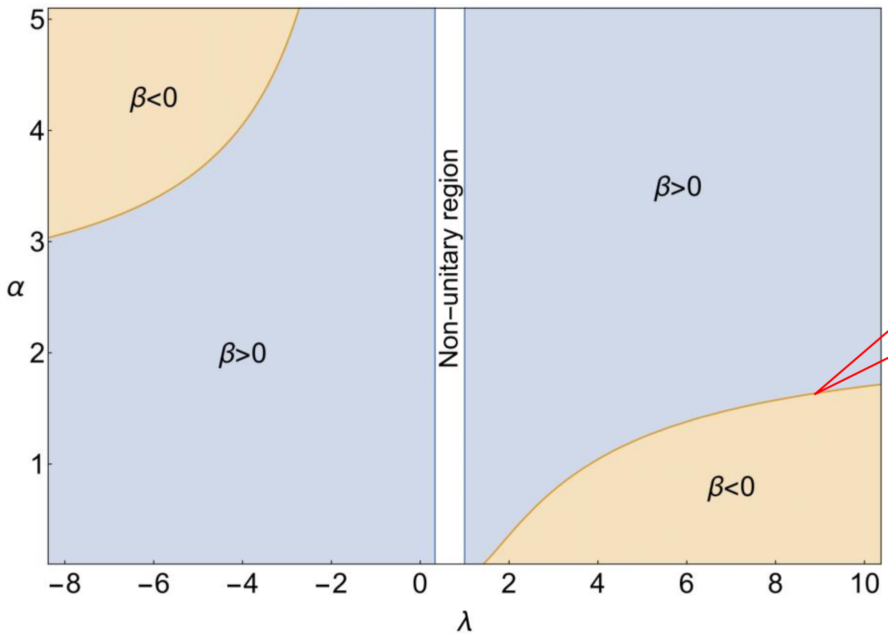
$$S = \frac{1}{2G}\int dt\, d^dx \sqrt{\gamma} N \Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}(\gamma)\Big)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \dots$$

$$\beta_G, \qquad \beta_\lambda$$

M. Herrero-Valea, S. Sibiryakov & A.B.,
PRD100 (2019) 026012

$$\alpha \equiv \frac{1}{\nu_5}\frac{(1-\lambda)(8\nu_4 + 3\nu_5)}{1-3\lambda}$$



$$\lambda(\alpha) = \frac{9 + 7\sqrt{\alpha} - 2\alpha + 2\sqrt{10(\alpha + \alpha^{3/2})}}{3(3 + \sqrt{\alpha} - 2\alpha)}$$

The lines $\lambda=\lambda(\alpha)$ correspond to potential location of fixed points of the full RG flow. The region $\lambda \in [1/3,1]$ is excluded by the requirement of unitarity.

Motivation for UMG $\rightarrow$ GUMG transition

Consistent QG in UV: GR $\rightarrow$ Horava gravity

Dark energy problem: GR $\rightarrow$ UMG $\rightarrow$ GUMG

Einstein (1919)

UMG $\downarrow$ Λ GUMG $\downarrow$ $p = w\varepsilon$

what is in common ?

violation of Lorentz and diffeomorphism symmetry

fixing the lapse function (projectable HL model)

same number of DOF (TT + scalar graviton)

. . .

Generalized unimodular gravity (GUMG)

Constraint on metric coefficients

$$(-g^{00})^{-1/2} = N(\gamma), \quad \gamma = \det \gamma_{ij}.$$

Unimodular gravity case

$$\det g_{\mu\nu} = -1 \Rightarrow N(\gamma) = \frac{1}{\sqrt{\gamma}}$$

$$S_{GUMG}[\,g_{ij}, g_{0i}\,] = S_{EH}[\,g_{\mu\nu}] \Big|_{(-g^{00})^{-1/2} = N(\gamma)}$$

Effective perfect fluid composed of the metric

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} T_{\mu\nu},$$

$$T_{\mu\nu} = (\varepsilon + p)\, u_\mu u_\nu + p\, g_{\mu\nu}, \quad u_\mu = -\delta^0_\mu N \qquad \varepsilon = 2\, u^\mu u^\nu \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, R\right)$$

Equation of state

$$p = w\varepsilon, \quad w = 2\,\frac{d \ln N(\gamma)}{d \ln \gamma}$$

Canonical formalism

$$S[\gamma_{ij}, \pi^{ij}, N^i, P_i, v^i] = \int dt\, d^3x \left(\pi^{ij}\dot{\gamma}_{ij} + P_i\dot{N}^i - NH_\perp - N^iH_i - v^iP_i\right)$$

Canonical constraints:

$$P_i = 0,$$
$$H_i = 0,$$
$$\partial_i T = 0, \qquad T \equiv wNH_\perp,$$
$$T\partial_i S = 0.$$

$$H_\perp = \frac{\gamma_{im}\gamma_{jn}\pi^{ij}\pi^{mn} - \frac{1}{2}\pi^2}{\sqrt{\gamma}} - \sqrt{\gamma}\,{}^3R$$
$$H_i = -2\gamma_{ij}\nabla_k\pi^{jk}, \quad \pi = \gamma_{ij}\pi^{ij},$$

$$S = \Omega\,\partial_k N^k - \frac{d\ln w}{d\ln\gamma}\frac{\pi N}{\sqrt{\gamma}},$$
$$\Omega = 1 + w + 2\frac{d\ln w}{d\ln\gamma}$$

Bifurcation:

$$T = 0 \qquad\qquad \partial_i S = 0.$$

GR branch — ***GUMG branch***

↓

three local degrees of freedom – tensor and scalar gravitons

No classical transitions between branches

$$\dot{T} = TS$$

Are they stable (non-ghost)?

GUMG cosmology

$$ds^2 = -N^2 dt^2 + a^2(t)\sigma_{ij} dx^i dx^j$$

$k = 0, \pm 1$

$$H = \frac{\dot{a}}{Na}$$ ***Hubble factor***

$$N = N(\gamma) = N(a^3)$$

Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{C}{3Na^3}, \qquad \varepsilon = \frac{M_P^2 C}{Na^3}$$ ***Dark fluid density***

$$\frac{d\varepsilon}{da} = -3\,(1+w)\,\frac{\varepsilon}{a}$$

Depending on the choice of $N(\gamma)$ can imitate dark energy *(without phantom line crossing)* and inflation stage

Reconstruction of the model from inflationary stage

$$N(\gamma) = \frac{1}{\sqrt{\gamma}}\left[1 + \sqrt{\frac{\gamma}{\gamma_*}} + B\left(\frac{\gamma}{\gamma_*}\right)^{3/2-2\Delta}\right], \qquad \Delta \sim 1 - n_s > 0$$

$$w \simeq -1 + \sqrt{\frac{\gamma}{\gamma_*}}$$

End of inflation:

$$\sqrt{\gamma} \to \sqrt{\gamma_*},$$

$$w \to 0$$

GUMG cosmological perturbation theory

$$\delta\gamma_{ij} = a^2(-2\psi\,\sigma_{ij} + 2\nabla_i\nabla_j E + 2\nabla_{(i}F_{j)} + t_{ij}),$$

Physical scalar sector -- tensor and scalar graviton

Scalar graviton = inflaton

$$S = \frac{1}{2}\int d\eta\, d^3x\Big(\vartheta'^2 + c_s^2\,\vartheta\Delta\vartheta + \frac{\theta''}{\theta}\vartheta^2\Big),$$

$$\vartheta = \theta\,\psi, \qquad \theta^2 = 3a^2 M_P^2\frac{\Omega}{w}$$

$\vartheta' = d\vartheta/d\eta$

$\eta = \int dt\, N/a$

conformal time

Unitarity domain – free from ghost and tachyon instabilities

$$\frac{w}{\Omega} > 0 \qquad 1 + w > 0$$

$$\Omega = 1 + w + 2\frac{d\ln w}{d\ln\gamma}$$

Speed of sound

$$c_s^2 = \frac{w(1+w)}{\Omega}.$$

No crossing the phantom divide line in unitarity domain

Inflation and its power spectra

Mukhanov-Sasaki equation

$$\vartheta'' - c_s^2\,\Delta\vartheta - \frac{\theta''}{\theta}\vartheta = 0$$

Power spectrum

$$\delta_\psi^2(k,\eta) = \frac{k^3}{2\pi^2}\frac{\left|\vartheta_{\mathbf{k}}(\eta)\right|^2}{\theta^2(\eta)}$$

Scalar sector

$$\delta_\psi^2(k,\eta) = \frac{1}{12\pi^2}\sqrt{\frac{\Omega}{w(1+w)^3}}\,\frac{H^2}{M_P^2}\bigg|_{c_s k = Ha} = \frac{1}{36\pi^2}\frac{1}{c_s(1+w)}\frac{\varepsilon}{M_P^4}\bigg|_{c_s k = Ha}$$

red tilt

$$n_s - 1 = \frac{-6(1+w) + \frac{d\ln\Omega}{d\ln a} - \frac{d\ln w}{d\ln a} - 3\frac{d\ln(1+w)}{d\ln a}}{-(1+3w) + \frac{d\ln\Omega}{d\ln a} - \frac{d\ln w}{d\ln a} - \frac{d\ln(1+w)}{d\ln a}}\Bigg|_{c_s k = Ha}$$

Standard result of k-inflation with a speed of sound c_s

Tensor sector

$$\delta_t^2(k,\eta) = \frac{2}{\pi^2}\frac{H^2}{M_P^2}\bigg|_{k=Ha}, \qquad n_t \simeq -3(1+w)\big|_{k=Ha}$$

$$r \equiv \frac{\delta_t^2}{\delta_\Phi^2} = 54\,\frac{H^2_{k=Ha}}{H^2_{c_s k = Ha}}\left[c_s(1+w)\big|_{c_s k = Ha}\right]$$

Parameters of power spectra in GUMG theory:

$$N(\gamma) = \frac{1}{\sqrt{\gamma}}\left[1 + \sqrt{\frac{\gamma}{\gamma_*}} + B\left(\frac{\gamma}{\gamma_*}\right)^{3/2-2\Delta}\right],$$

$$w \simeq -1 + \sqrt{\frac{\gamma}{\gamma_*}}$$

$$\delta^2_\Phi(k,\eta) \simeq \frac{\sqrt{6B}}{27\pi^2}\frac{H_0^2}{M_P^2}\left(\frac{\gamma}{\gamma_*}\right)^{-\Delta}\Big|_{c_s k = Ha},$$

$$n_s \simeq 1 - \frac{3}{2}\Delta, \qquad r \simeq \frac{54}{\sqrt{6B}}\left(\frac{\gamma}{\gamma_*}\right)^{\Delta}\Big|_{c_s k = Ha}$$

$$1 - n_s \simeq 0.04, \quad \delta^2_\Phi \simeq 10^{-10}, \quad r \ll 1$$

$$\frac{\gamma_*}{\gamma}\Big|_{c_s k = Ha} = e^{6\mathcal{N}}, \quad \mathcal{N} \simeq 60$$

exponentially high e-folding number

Naturalness $B = O(1),$

$$r \sim 54\left(e^{-\mathcal{N}}\right)^{4(1-n_s)} \sim 10^{-3}$$

GUMG covariantization and self-gravitating media

Introducing Stueckelberg fields

$$g^{\mu\nu} \mapsto C^{AB}, \quad C^{AB} = g^{\mu\nu}\partial_\mu \phi^A \partial_\nu \phi^B, \quad \phi^A = (\phi^0, \phi^1, \phi^2, \phi^3)$$

New form of the constraint

$$(-g^{00})^{-1/2} - N(\gamma) = 0 \quad \Rightarrow \quad P(X) - \sqrt{\det C^{AB}} = 0$$

$$X = C^{00} = g^{\mu\nu}\partial_\mu \phi^0 \partial_\nu \phi^0$$

kinetic term for ϕ^0

$$P(X) \equiv \left(\frac{-X}{\Gamma(1/\sqrt{-X})}\right)^{1/2}, \quad \Gamma(N(\gamma)) \equiv \gamma$$

Inverse function to $N(\gamma)$

Constraint with the Lagrangian multiplier

$$S[g_{\mu\nu}, \Lambda, \phi^A] = S_{EH}[g_{\mu\nu}] + \int d^4x \sqrt{-g}\, \Lambda \left(P(X) - \sqrt{\det C^{AB}}\right)$$

$$\Updownarrow$$

Self-gravitating media action

$$S[g_{\mu\nu}, \Lambda, \phi^A] = S_{EH}[g_{\mu\nu}] + \int d^4x \sqrt{-g}\, U(C^{AB}(g_{\mu\nu}))$$

Decoupling of spatial Stueckelbergs:

EoM: $$\Lambda = K(\phi^0) \equiv K(\phi), \quad \phi^0 \equiv \phi$$

$$\Downarrow$$

Special type of k-essence:

$$S_K[g_{\mu\nu}, \phi] = S_{EH}[g_{\mu\nu}] + \int d^4x\sqrt{-g}\, K(\phi)P(X)$$

$$p = w\rho = \Lambda P, \quad w = \frac{P}{2P_X X - P}$$

Hydrodynamical nature of the speed of sound

$$c_s^2 = \frac{\partial p/\partial X}{\partial \rho/\partial X} = \frac{\partial_\gamma P}{\partial_\gamma (P/w)} = \frac{w(1+w)}{\Omega}$$

N. Kolganov, A. Vikman
& A.B. e-Print: 2011.06521

Reconstruction of $K(\phi)$ in GUMG k-inflation

H_0 -- Hubble scale of inflation

Expansion in two smallness parameters

$$\delta = \sqrt{\frac{-\nabla^\mu\phi\nabla_\mu\phi}{\gamma_*}} \ll 1, \quad \varepsilon = \frac{3H_0\phi}{\sqrt{\gamma_*}} \equiv \frac{\phi}{\phi_0} \ll 1$$

$\gamma_* = a_*^3$ *-- scale factor at the end of inflation*

$$P(X) = 1 - \delta - B\,\delta^{3+8\frac{n_s-1}{3}} + \dots,$$

$$K(\phi) = -3M_P^2H_0^2\left[1 - \varepsilon + \frac{3}{4}\varepsilon^2 - \frac{1}{2}\varepsilon^3 - \left(3 + 8\,\frac{n_s-1}{3}\right)B\,\varepsilon^{3+8\frac{n_s-1}{3}} + \dots\right]$$

k-essense Lagrangian

$$\mathcal{L}_K(\phi, X) = \frac{3M_P^2H_0^2}{\sqrt{\gamma_*}}\left(\sqrt{-\nabla_\mu\phi\nabla^\mu\phi} - \sqrt{\gamma_*}\right) + O\left(\varepsilon, \varepsilon^{3+8\frac{n_s-1}{3}}, \delta, \delta^{3+8\frac{n_s-1}{3}}\right)$$

Cuscuton model (no degrees of freedom)

Dynamical k-inflaton is sitting here

$$\Delta = \frac{2}{3}(1 - n_s) \simeq 0.026 \ll 1$$

$$\varepsilon^{8\frac{n_s-1}{3}} \simeq 1 - 4\Delta \ln\frac{\phi}{\phi_0},$$

$$\delta^{8\frac{n_s-1}{3}} \simeq 1 - 2\Delta \ln\frac{-\nabla_\mu\phi\nabla^\mu\phi}{\gamma_*}$$

EFT type slowly varying logarithmic corrections

Unifying GUMG and Horava-Lifshitz gravity: generalized RG?

$$g_A = (\lambda, \Lambda, \eta, \mu_1, \mu_2, \nu_1, \ldots) \to g_A(\gamma) = (\lambda(\gamma), \Lambda(\gamma), \eta(\gamma), \mu_1(\gamma), \mu_2(\gamma), \nu_1(\gamma), \ldots)$$

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma} N(\gamma)\Big(K_{ij}K^{ij} - \lambda(\gamma)K^2 - \mathcal{V}(\gamma)\Big)$$

$$\mathcal{V}(\gamma) = 2\Lambda(\gamma) - \eta(\gamma)R + \mu_1(\gamma)R^2 + \mu_2(\gamma)R_{ij}R^{ij} + \nu_1(\gamma)R^3 + \nu_2(\gamma)RR_{ij}R^{ij}$$
$$+\nu_3(\gamma)R^i_j R^j_k R^k_i + \nu_4(\gamma)\nabla_i R\nabla^i R + \nu_5(\gamma)\nabla_i R_{jk}\nabla^i R^{jk} + \ldots$$

Generalized RG

$$\partial_t g_A = \beta_A(g, \partial_\gamma g, \partial^2_\gamma g, \ldots)$$

A.Yu.Kamenshchik, I.Karmazin & A.B. Phys.Rev. D 48 (1993) 3677, gr-qc/9302007 ;
D.I. Kazakov, Teor. Mat. Fiz. 75, 157 (1988)

Conclusions

Renormalization of Horava-Lifshitz gravity

Salvation of unitarity in local renormalizable QG via LI violation
BPHZ renormalization and "regularity" of propagators
Gauge invariance of UV counterterms
Asymptotic freedom in (2+1)-dimensional theory
Towards (3+1)-dimensions

Generalized unimodular gravity (GUMG)

Dark fluid with barotropic time dependent equation of state
GUMG cosmology and unitarity domain for a scalar graviton
Inflationary scenario and power spectra (naturalness)
Covariantization, decoupling of spatial Stueckelberg fields
k-essence and self-gravitating media, EFT

THANK YOU!