Remarks on nonsingular black holes

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Famous Penrose and Hawking theorems on singularities inside black holes imply that the General Relativity does not give consistent description of the spacetime in the BH interior.

Stationary BH solutions of the Einstein equations have a curvature singularity in their interior.

For Schwarzschild BH $\mathcal{K} = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \dfrac{48M^2}{r^6}.$

Singularities exist both in cosmology and inside BHs

- In cosmology, where $C^2 = 0$, and in order to restrict infinite growing of rhe curvature it is sufficient to modify the equation of state ($T_{\alpha\beta}$);
- For black holes this is not sufficient: main problem is fast growing anisotropy (C^2).

Proper time: $\tau \approx \frac{2}{3\sqrt{2M}} r^{3/2}.$

$ds^2 \sim -d\tau^2 + a\,\tau^{-2/3} dt^2 + b\,\tau^{4/3} d\omega^2$

Contracting Kasner-type anisotropic universe.

$S^2 \times R$

How to prevent curvature growing?

- Particle creation

Modified gravity: Options

(i) Modified fundamental gravity (higher derivatives, $f(R)$ theory, etc.);

(ii) Nonlocal modification (Ghost-free gravity);

(iii) Gravity as an emergent phenomenon (strings, loops, etc.): Classical ST terminates its existence. New phase.

Phenomenological approach

(i) There exists a fundamental length scale $\ell >> l_{Pl}$;

(ii) In the domain where $\Re \ll \ell^{-2}$ the metric obeys the Einstein equations with small corrections;

(iii) In the domain where $\Re \sim \ell^{-2}$ the Einstein equations should be modified;

(iv) Limiting curvature condition (LCC): $|\Re| \leq \frac{C}{\ell^2}$. C is a dimensionless universal constant, defined by the theory and independent of the parameters of the solution. [Markov, JETP Lett. 36, 265 (1982)]

Main goal is to study such NSBH models and try to find their robust predictions.

Quadratic in curvature invariants:

$$C^2 = C_{\alpha\beta\chi\delta}C^{\alpha\beta\chi\delta}, \qquad R^2,$$

$$S^2 = S_{\alpha\beta}S^{\alpha\beta}, \quad S_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{4}g_{\alpha\beta}R.$$

$$\mathcal{K} = R_{\alpha\beta\chi\delta}R^{\alpha\beta\chi\delta} = C^2 + 2S^2 + \frac{1}{6}R^2.$$

$$S^2 = 0 \rightarrow \; G_{\alpha\beta} = \frac{1}{4}g_{\alpha\beta}R.$$

General form of SS metric in advanced time coordinates

$ds^2 = -\alpha^2 f\, dv^2 + 2\alpha\, dv\, dr + r^2 d\Omega^2, \quad f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$.

$f\,|_{r\to\infty} = \alpha\,|_{r\to\infty} = 1$. In a static ST: $\xi_{\mathrm{t}}^2 = -\alpha^2 f$,

Apparent horizon: $f = 0$.

$\mathscr{K} \sim 4\left(\frac{[f-1]}{r^2}\right)^2 \Rightarrow$ If ST is regular at $r = 0$,

the apparent horizon cannot cross this line.

$f(v,r) = 1 + f_2(v) r^2 + \ldots, \quad \alpha(v,r) = \alpha_0(v) + \alpha_2(v) r^2 + \ldots$

Nonsingular BH: (Quadratic) curvature invariants are finite + LCC is valid.

Static nonsingular BHs.

A static SSBH with α=1 and $f = \dfrac{P_n(r)}{Q_n(r)}$.

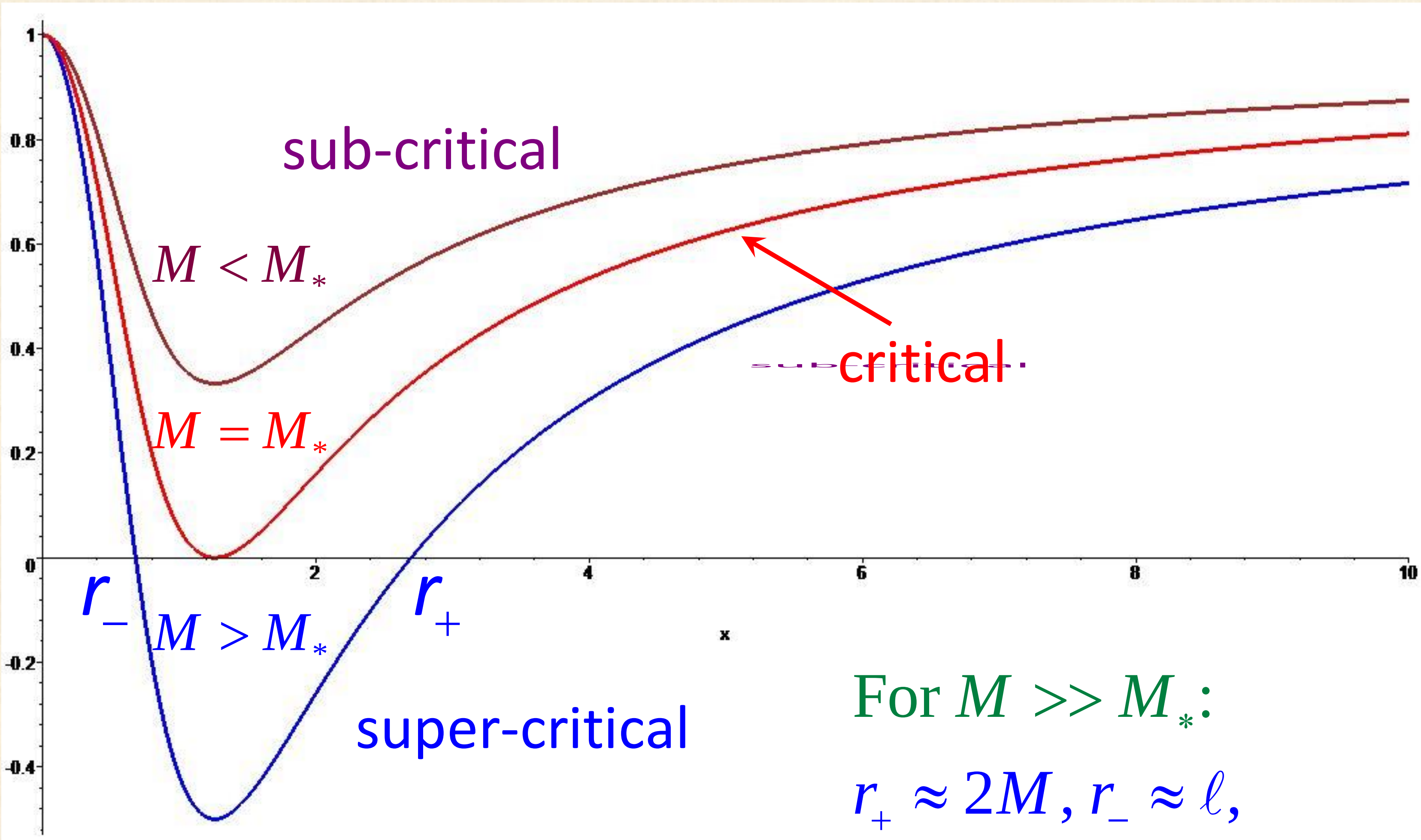
Metrics with $n \le 2$ cannot be consistent metrics for a nonsingular black hole. [V.F. PR D94,104056 (2016)].

$n = 3$ example: Hayward metric ('06):

$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}.\ M = \frac{m^3}{2\ell^2},\ r = mx,$$

$$f = 1 - \beta^{-1}\frac{x^2}{1 + x^3},\quad \beta = \left(\frac{\ell}{2M}\right)^{2/3}.$$

$$M_* = \frac{3\sqrt{3}}{4}\ell$$



For $M >> M_*$:

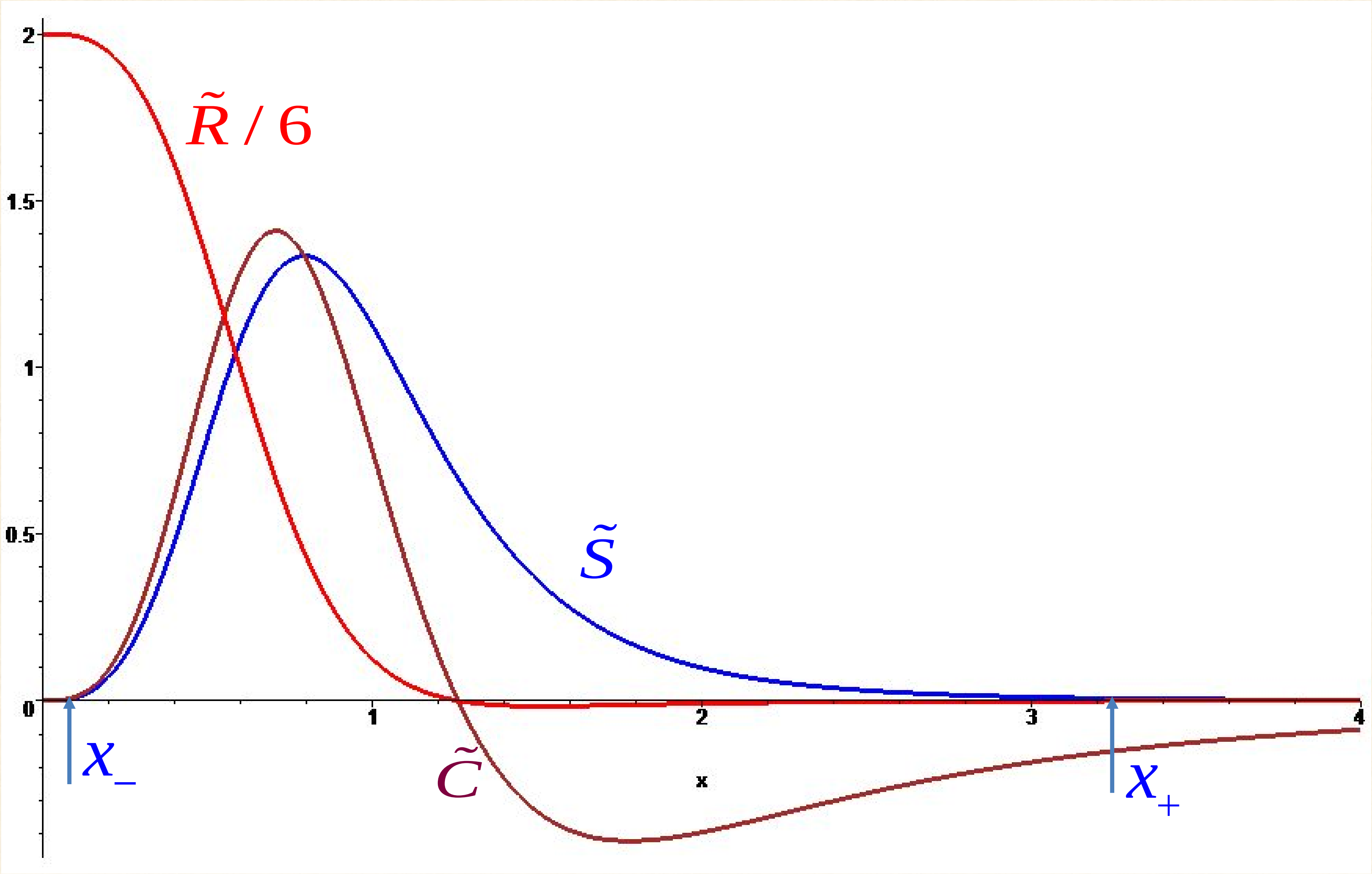
$r_+ \approx 2M, \; r_- \approx \ell,$

$x_- \approx \beta^{1/2}, \; x_+ \approx \beta^{-1}$

Mass-gap for mini BHs formation is a generic feature for NS BH models.

$$\tilde{C} = \pm\sqrt{3C^2}\,\ell^2,\ \tilde{R} = R\ell^2,\ \tilde{S} = \sqrt{S^2}\,\ell^2.$$

- These quantities depend on x only;
- These expressons remain the same even if $M = M(v)$.



$\tilde{R}/6$
$\tilde{S}$
$\tilde{C}$
X_-
X_+
x
2
1.5
1
0.5
0
1
2
3
4

Hayward metric satisfies LCC;

DeSitter core at the center:

$$C^2 = S^2 = 0,\ R \sim \ell^{-2} = const$$

Hawking out-going radiation in the BH exterior is accompanied by the in-coming energy fluxes in the BH interior. One can approximate these fluxes by the in-coming null fluid.

Spherically symmetric null fluid in in a static SS gravitational field.

$$ds^2 = -f(r)\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2.$$

$$du = dv - 2\frac{dr}{f},$$

$u =$ const and v $=$ const are null surfaces.

$$l_\mu = v_{,\mu}, \quad n_\mu = u_{,\mu}.$$

In-coming flux: $T^{in}_{\mu\nu} = \rho\, l_\mu l_\nu$;

Out-going flux: $T^{in}_{\mu\nu} = \rho\, n_\mu n_\nu$.

Conservation law: $T_{\mu\nu}{}^{;\nu} = 0 \rightarrow \rho = \frac{\varepsilon}{4\pi r^2}.$

$$\mathfrak{I}^{in} = T_{\mu\nu}^{in} T^{in,\mu\nu} = 0, \quad \mathfrak{I}^{out} = T_{\mu\nu}^{out} T^{out,\mu\nu} = 0,$$

$$T_{\mu\nu} = T_{\mu\nu}^{in} + T_{\mu\nu}^{out} = \rho(l_\mu l_\nu + n_\mu n_\nu).$$

$$\mathfrak{I} = T_{\mu\nu} T^{\mu\nu} = \frac{8\rho^2}{f^2}.$$

In GR: a backreaction of $T_{\mu\nu}^{in}$ results in Vaidya solution: $M \to M(v)$. Null fluid matter is absorbed by a singularity, and no out-going null fluid: $\mathfrak{T} = \mathfrak{T}^{in} = 0$.

Nonsingular BH: Inside the inner horizon the test null fluid has both in- and out-components.

- Near inner horizon a divergence : $\mathcal{T} = \frac{8\rho^2}{f^2}$.
- Test field approximation does not work.
- LCC can be violated.
- Mass inflation (Israel, Poisson,'89).

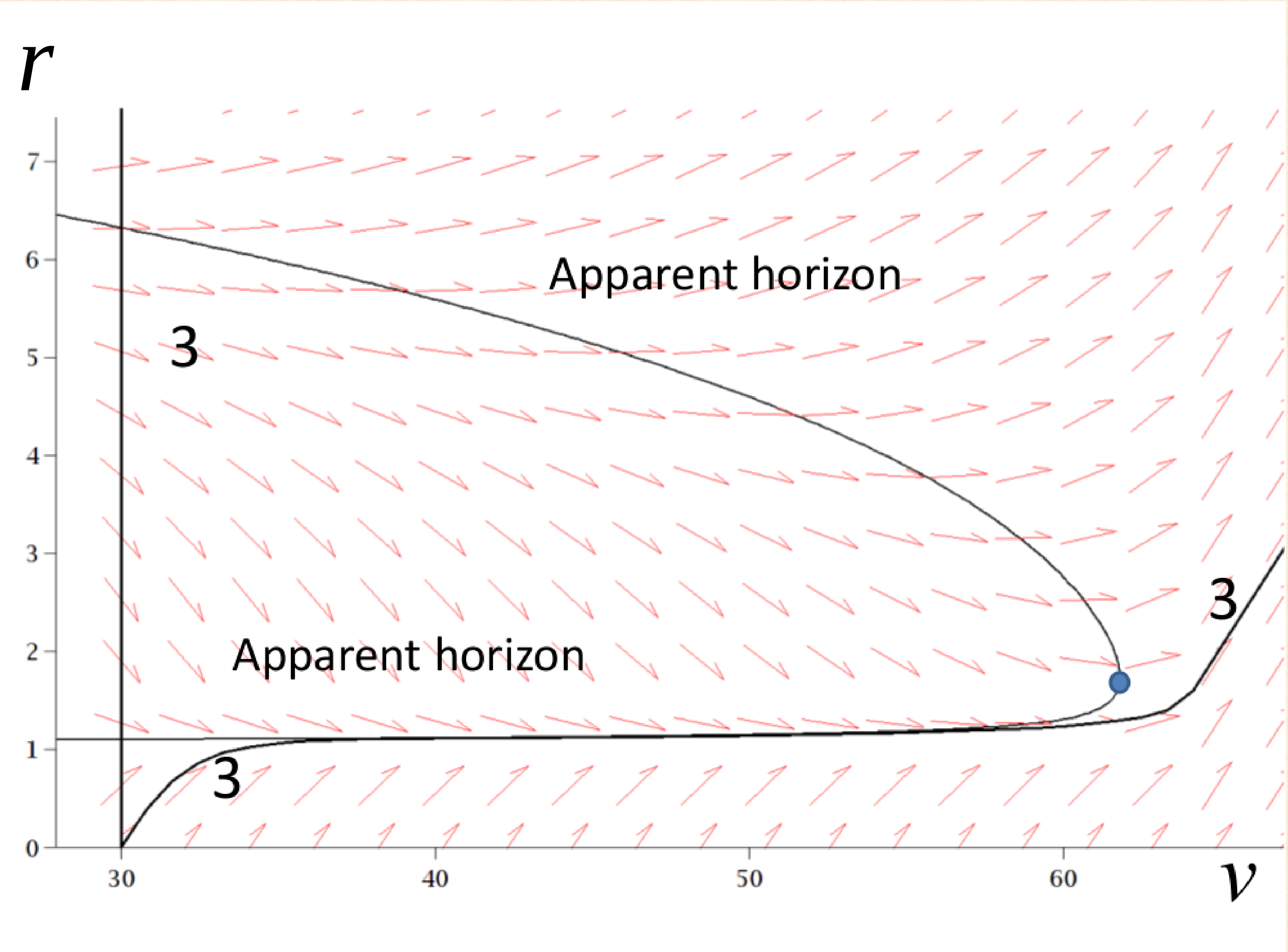
What happens in Hayward model with in-coming null fluid? $M \to M(v)$.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \quad \mathcal{G} = G_{\mu\nu} G^{\mu\nu}.$$

- $\ell^4 \, \mathcal{G}$ depends on x only;
- $\mathcal{G}$ is uniformly bounded;
- No out-going flux.

HBH: The core is a "condensate" with density $\sim \ell^{-2}$ and deSitter like equation of state. It totally absorbes in-coming radiation, and namely this makes it possible the validity of *LCC*.

Evaporating NS black hole model


r
7
6
5
4
3
2
1
0
30
40
50
60
v
Apparent horizon
Apparent horizon
3
3
3

For complete evaporation:

- The apparent horizon is closed and has "O-shape" on (v, r) plane. Its topology in 4D ST is $S^1 \times S^2$ (torus);
- No event horizon and no black hole (in its exact mathemaical sence);
- No loss of information;
- It returns back after a time-delay determined by the evaporation time.

During all the evolution of v-dependent Hayward metric the quadratic curvature invariants remain uniformly bounded and the metric obeys LCC.

(i) An apparent horizon in a regular metric cannot cross $r = 0$.

(ii) It has two branches: outer- and inner-horizons.

(iii) Non-singular BH model with a closed apparent horizon

[V.F. and G.Vilkovisky, Phys. Lett., 106B, 307 (1981)]

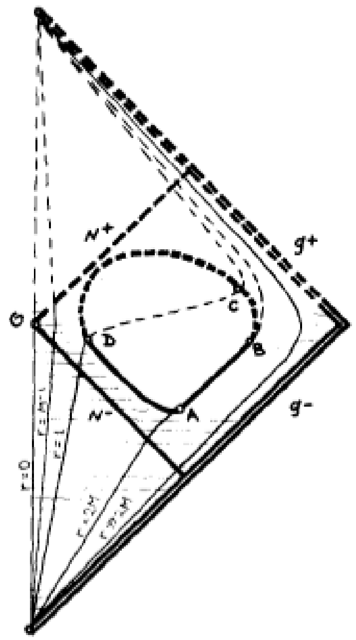


Fig. 1. Penrose diagram for the collapse of the null shell ($M \gg 1$). Solid (dashed) lines are used for the known (hypothetical) details of the picture. The shaded region is the region of validity of the obtained asymptotic solution. The line $N^- \cup N^+$ is the world line of the null shell. The closed and dashed bold line $ABCD$ is the apparent horizon. The light lines are the level lines r = const.

Near-threshold metric

$$M_* = \frac{3\sqrt{3}}{4}\ell,\ r_* = \sqrt{3}\ell: \quad f(r_*) = f'(r_*) = 0.$$

In the vicinity of this point:

$$M = M_*(1+\mu(v)), \quad r = r_*(1+\rho),$$

Apparent horizon: $f(r_+) = 0 \to$

$$\rho_+(v) = \sqrt{\frac{2}{3}}\sqrt{\mu(v)}.$$

Surface gravity: $\kappa = \frac{1}{2} f'(r_+) = \frac{\sqrt{2}}{3\ell}\sqrt{\mu(v)}.$

To estimate the rate of change of mass at the final stage of the evaporation we use arelation

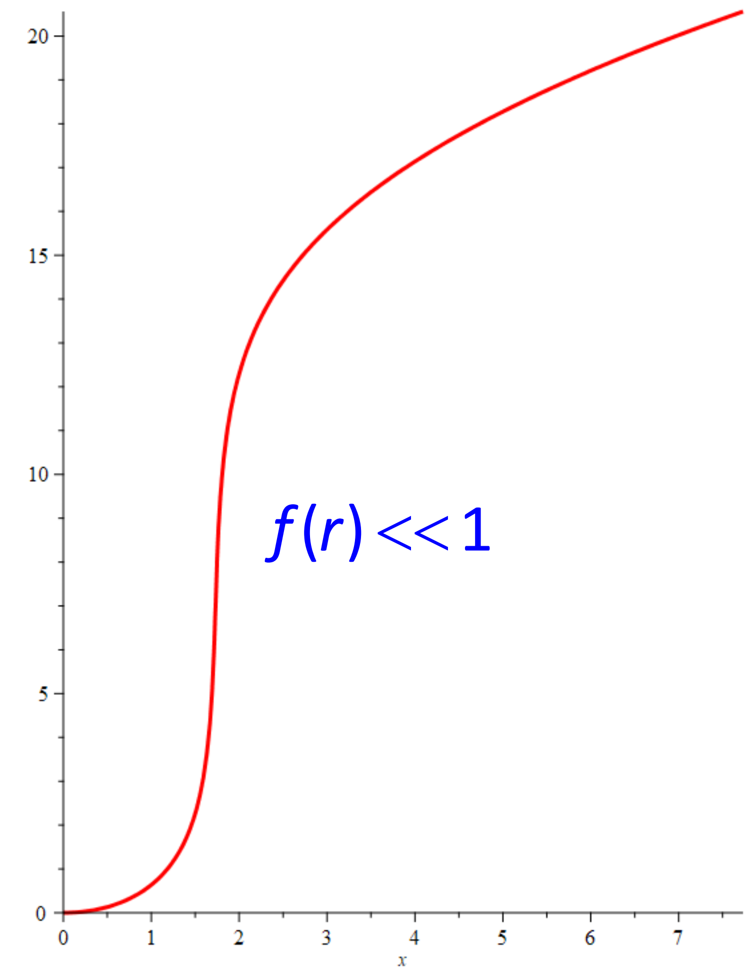
$$\frac{dM}{dv} = -\beta l_{Pl}^2 A_{BH} \theta^4 , \quad \theta = \frac{\kappa}{2\pi}.$$

$$\frac{d\mu}{d(v/\ell)} \sim -\frac{l_{Pl}^2}{\ell^2}\mu^2 , \quad \mu(v) = \frac{\mu_0}{1 + \frac{l_{Pl}^2}{\ell^3}\mu_0 v}.$$

Formally time of BH evaporation is infinite.

More realistic: $T \sim M^4 >> M^3$.

BH remnant (geon-type)



$f(r) << 1$

- Embedding diagram for a slice t=const of the subcritial metric $\mu < 0, \quad |\mu| << 1.$

$$\mathrm{dl}^2 = \frac{dr^2}{f(r)} + r^2 d\omega^2,$$

Matter content

Let Σ be a timelike $(t = \text{const})$ surface in a static ST

and $u^\mu = \dfrac{\xi^\mu}{\sqrt{-\xi^2}}$ be a unit vector orthogonal to it.

Proper mass density $\varepsilon = "T_{\mu\nu}" u^\mu u^\nu = \dfrac{1}{8\pi} G_{\mu\nu} u^\mu u^\nu$.

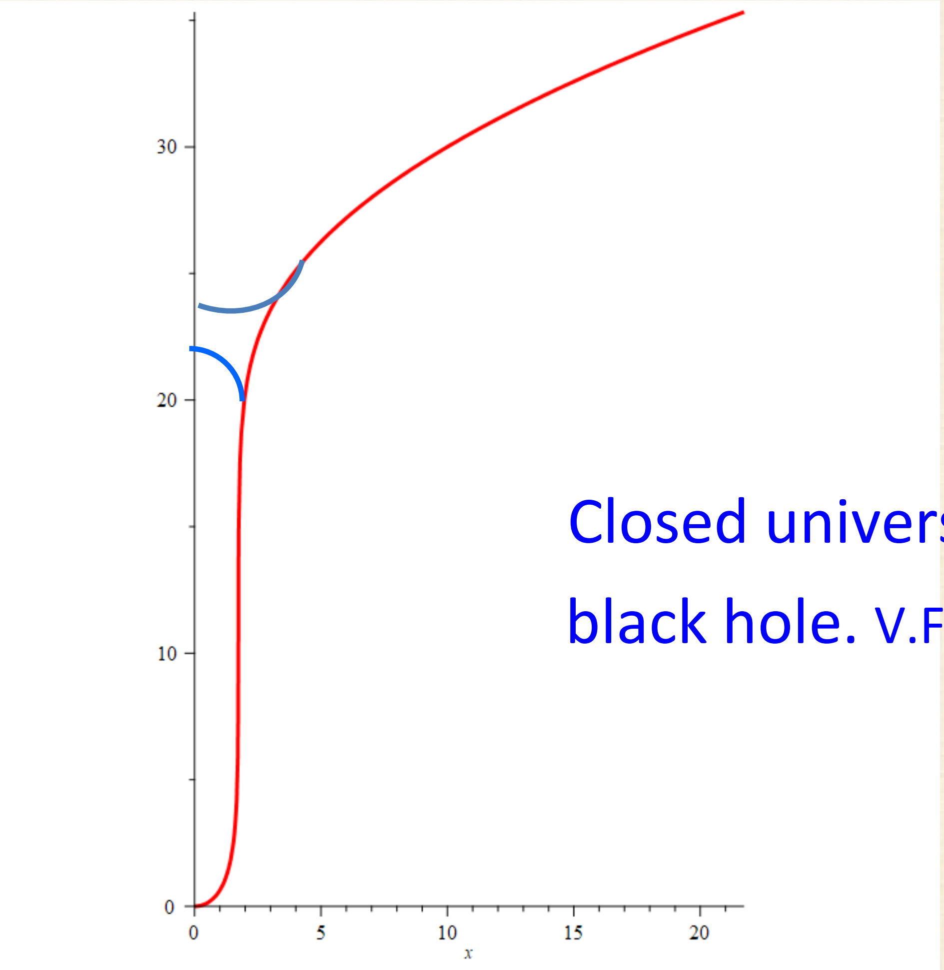
Slightly subcritial metric, $\mu<0, |\mu| << 1$.

Mass at infinity: $M = -\frac{1}{2}\int_0^\infty G_0^0 r^2 dr.$

Proper mass $\mathcal{M} = -\frac{1}{2}\int_0^\infty G_0^0 \frac{r^2 dr}{\sqrt{f(r)}} \sim -M_* \ln|\mu|.$

Final stage:

- Complete evaporation. BH life-time $M^4 >> M^3$?
- New universe formation inside BH?



Closed universe formation inside the black hole. V.F., Markov, and Mukhanov ('89,'90)

Instead of summary

Robust properties of nonsigular black holes:

- Existence of inner branch of the apparent horizon;
- For completely evaporating BH the apparent horizon has torus-like topology.
- Mass gap for mini black holes;

Example: Hayward model of a nonsingular black hole (LCC is valid):

- Absorbing DeSitter-like core;
- No mass inflation;
- Geon-type remnant.

Open question: What are effective modified gravity equations which have non-singular BH solutions?