# Geometric inflationary models based on higher-derivative gravity

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS, Moscow - Chernogolovka, Russia

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# The higher-derivative model without ghosts

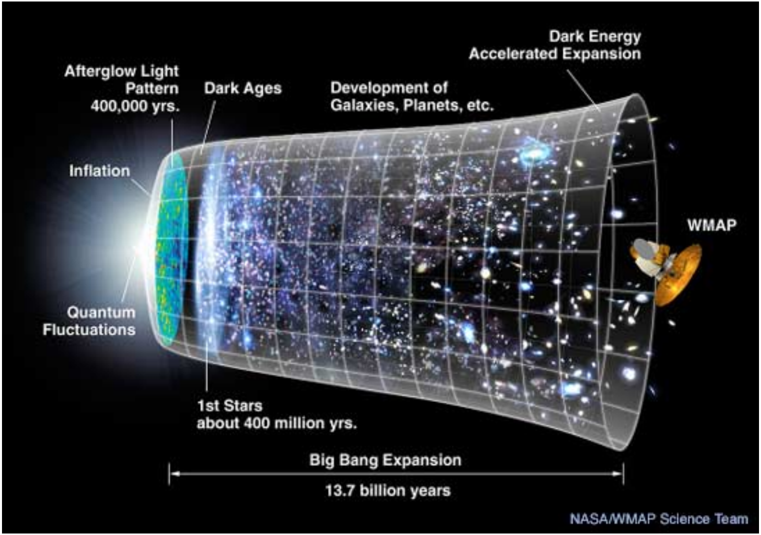
The specific type of $f(R)$ gravity in metric formalism:

$$S = \int \left( \frac{R}{16\pi G} + A(R)R^2 \right) \sqrt{-g}\, d^4x$$

with $A(R) > 0$ slowly depending on $R$
$(|A'| \ll AR^{-1},\ |A''| \ll AR^{-2})$.

1. Why using $f(R)$ gravity (and in metric formalism):
a) to have a scalar gravitational degree of freedom;
b) to follow the Einstein's way of geometrization of physics as much as possible, at least in the approximate form.

2. Why using the correction to GR close to $R^2$?
a) From the classical side, such model likes de Sitter space-times (in the absence of the Einstein term and $A = \mathrm{const}$, DS or ADS with any curvature is a solution).
b) From the quantum side, $A(R)$ is dimensionless.


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

# Inflation

The (minimal variant of the) inflationary scenario is based on the two cornerstone independent ideas (hypothesis):

1. Existence of inflation (or, quasi-de Sitter stage) – a stage of accelerated, close to exponential expansion of our Universe in the past preceding the hot Big Bang with decelerated, power-law expansion.

2. The origin of all inhomogeneities in the present Universe is the effect of gravitational creation of pairs of particles - antiparticles and field fluctuations during inflation from the adiabatic vacuum (no-particle) state for Fourier modes covering all observable range of scales (and possibly somewhat beyond).

NB. This effect is the same as particle creation by black holes, but no problems with the loss of information, 'firewalls', trans-Planckian energy etc. in cosmology, as far as observational predictions are calculated.

Both assumptions can be combined into one:
Some part of the world which includes all its presently observable part was as much symmetric as possible during some period in the past - both with respect to the geometrical background and to the state of all quantum fields (no particles).

Non-universal (due to the specific initial condition) explanation of the cosmological arrow to time - chaos, "entropy" (in some not well defined sense) can only grow after inflation.
Still this state is an intermediate attractor for a set of pre-inflationary initial conditions with a non-zero measure.

Remark regarding these initial conditions for perturbations: they are *not* in the Bunch-Davies state in the rigorous sense, since this state may not be imposed for arbitrary large scales. As a consequence, inflationary models typically does *not* predict regular behaviour at spatial infinity both during and after inflation ("multiverse").

Existing analogies in other areas of physics.

1. The present dark energy, though the required degree of metastability for the primordial dark energy is much more than is proved for the present one (more than 60 e-folds vs. $\sim 3$).

2. Creation of electrons and positrons in an external electric field.

# Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\mathcal{R}(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

$\mathcal{R}$ describes primordial scalar perturbations, $g$ – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

$$n_s(k) - 1 \equiv \frac{d \ln P_{\mathcal{R}}(k)}{d \ln k}, \quad r(k) \equiv \frac{P_g}{P_{\mathcal{R}}}$$

In fact, metric perturbations $h_{lm}$ are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\mathcal{R},\ g$).
In particular:

$$\hat{\mathcal{R}}_k = \mathcal{R}_k\, i(\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}}) + \mathcal{O}\left((\hat{a}_{\mathbf{k}} - \hat{a}^\dagger_{\mathbf{k}})^2\right) + \ldots + \mathcal{O}(10^{-100})(\hat{a}_{\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}}) + ,,,$$

The last non-commutative term is time dependent, it is affected by physical decoherence and may become larger, but not as large as the second term.
Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

All these predictions are beyond semiclassical gravity!

Semiclassical gravity: space-time metric $g_{ik}$ is not quantized and

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu R\right)(g_{ik}) = \left\langle \hat{T}_\mu^\nu(g_{ik})\right\rangle$$

Instead,

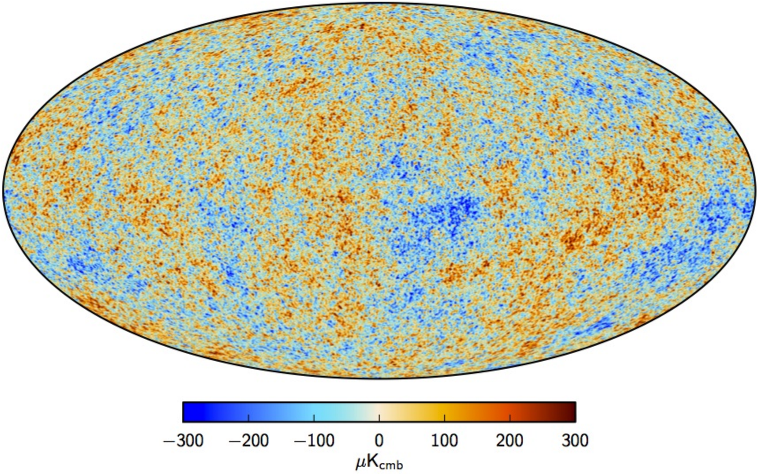
$$\frac{1}{8\pi G}\left(\hat{R}_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu \hat{R}\right)(\hat{g}_{ik}) = \hat{T}_\mu^\nu$$

is used.

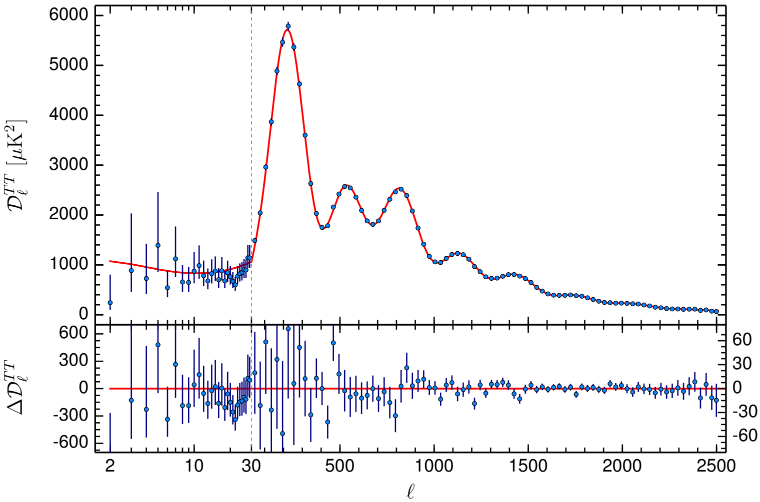
$\langle\mathcal{R}\rangle = 0$ does not mean the absence of perturbations.

# CMB temperature anisotropy

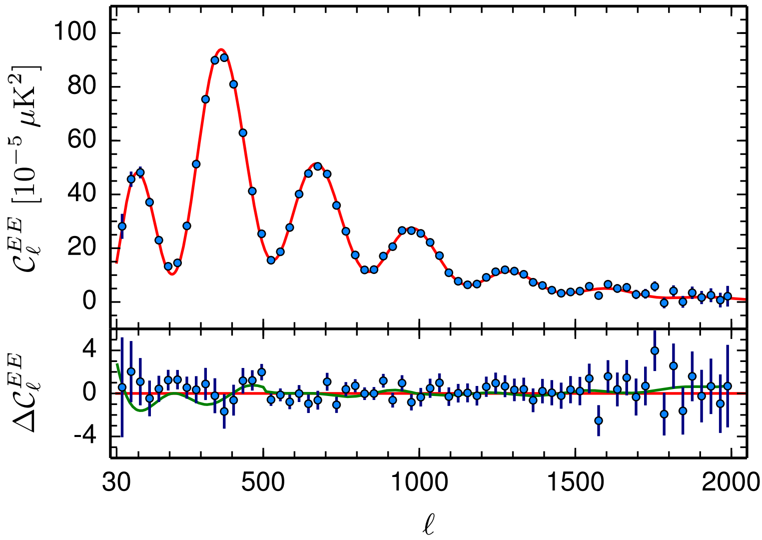
Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



# CMB temperature anisotropy multipoles



# CMB E-mode polarization multipoles



# New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) >= \int \frac{P_{\mathcal{R}}(k)}{k}\, dk, \;\; P_{\mathcal{R}}(k) = (2.10 \pm 0.03)\cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \;\; n_s - 1 = -0.035 \pm 0.004, \;\; r < 0.065$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s)N_H \sim 2$).

# The simplest models producing the observed $n_s$

1. The $R + R^2$ model (Starobinsky, 1980):

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \quad f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 3.1 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$N = \ln\frac{k_f}{k} = \ln\frac{T_\gamma}{k} - \mathcal{O}(10), \quad H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

2. The same prediction for a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large $\phi$ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0,\ |\xi| \gg 1$ (Spokoiny, 1984), including the Higgs inflationary model (Bezrukov and Shaposhnikov, 2008).

# The simplest purely geometrical inflationary model

$$\mathcal{L} = \frac{R}{16\pi G} + \frac{N^2}{288\pi^2 P_{\mathcal{R}}(k)} R^2 + (\text{small rad. corr.})$$

$$= \frac{R}{16\pi G} + 5.1 \times 10^8\, R^2 + (\text{small rad. corr.})$$

The quantum effect of creation of particles and field fluctuations works twice in this model:

a) at super-Hubble scales during inflation, to generate space-time metric fluctuations;

b) at small scales after inflation, to provide scalaron decay into pairs of matter particles and antiparticles (AS, 1980, 1981).

Weak dependence of the time $t_r$ when the radiation dominated stage begins:

$$N(k) \approx N_H + \ln \frac{a_0 H_0}{k} - \frac{1}{3} \ln \frac{M_{\rm Pl}}{M} - \frac{1}{6} \ln (M_{\rm Pl} t_r), \;\; M_{\rm Pl} = G^{-1/2}.$$

# Evolution of the $R + R^2$ model

1. During inflation ($H \gg M$):

$$H = \frac{M^2}{6}(t_f - t) + \frac{1}{6(t_f - t)} + ..., \quad |\dot{H}| \ll H^2$$

.

(for the derivation of the second term in the rhs - see A. S. Koshelev et al., JHEP 1611 (2016) 067).

2. After inflation ($H \ll M$):

$$a(t) \propto t^{2/3}\left(1 + \frac{2}{3Mt}\sin M(t - t_1)\right)$$

The most effective decay channel: into minimally coupled scalars with $m \ll M$. Then the formula

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}\,n_s) = \frac{R^2}{576\pi}$$

(Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977)) can be used for simplicity, but the full integral-differential system of equations for the Bogoliubov $\alpha_k, \beta_k$ coefficients and the average EMT was in fact solved in AS (1981). Scalaron decay into graviton pairs is suppressed (A. A. Starobinsky, JETP Lett. 34, 438 (1981)).

For this channel of the scalaron decay:

$$\Gamma = \frac{GM^3}{24}, \quad N(k) \approx N_H + \ln\frac{a_0 H_0}{k} + \frac{5}{6}\ln(G^{1/2}M)$$

that gives $N(k = 0.002\,\mathrm{Mpc}^{-1}) \approx 54$. For the Higgs and the mixed $R^2$-Higgs models, $N(k = 0.002\,\mathrm{Mpc}^{-1}) \approx 58$, the increase is mainly due to the large Higgs non-minimal coupling.

# Scalaron decay in more details

The trial scale factor behaviour:

$$a(t) = r(t)\left[1 + \frac{\psi(t)}{r^{3/2}(t)}\sin M\left(t - t_1 + \int^t \delta(\tilde{t})\,d\tilde{t}\right)\right]$$

Light particles creation by this scale factor occurs mainly at $r(t) = 2k/M$. The main contribution to their energy density is

$$\rho_m = \frac{1}{2\pi^2 r^4}\int_0^{Mr(t)/2} k^3|\beta_k(\infty)|^2 dk,$$

$$\frac{1}{r^4}\frac{d}{dt}\left(\rho_m r^4\right) = \Gamma\rho_M, \quad \rho_M = Mn_M \equiv \frac{3\psi^2 M^2}{8\pi G r^3}.$$

The second step: taking back reaction of created particles into account. Consider the exact trace equation

$$R + \frac{1}{M^2}\Box R = 8\pi G < T >_m,$$

$$6\left(\frac{\ddot r}{r} + \left(\frac{\dot r}{r}\right)^2\right) - \frac{3\psi^2 M^2}{r^3} - \frac{12\dot\psi M}{r^{3/2}} \cos M(t-t_1) +$$

$$+ \frac{12\delta\psi M^2}{r^{3/2}} \sin M(t-t_1) + ... = 8\pi G < T >_m .$$

By expressing $< T >_m$ through $\beta_k(t)$, keeping only first order terms, and equating the terms having the $\cos M(t-t_1)$ time dependence, we get:

$$\dot\psi + \frac{\Gamma\psi}{2} = 0.$$

$\delta$ is determined by equating terms $\propto \sin M(t-t_1)$.

Possible microscopic origins of this phenomenological model.

1. Follow the purely geometrical approach and consider it as the specific case of the fourth order gravity in 4D

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + (\text{small rad. corr.})$$

for which $A \gg 1,\ A \gg |B|$. Emergence of hierarchy! Approximate scale (dilaton) invariance and absence of ghosts in the curvature regime $A^{-2} \ll (RR)/M_P^4 \ll B^{-2}$.

One-loop quantum-gravitational corrections are small (their imaginary parts are just the predicted spectra of scalar and tensor perturbations), non-local and qualitatively have the same structure modulo logarithmic dependence on curvature.

2. Another, completely different way:

consider the $R + R^2$ model as an approximate description of GR + a non-minimally coupled scalar field with a large negative coupling $\xi$ ($\xi_{conf} = \frac{1}{6}$) in the gravity sector::

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{\xi R \phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \ .$$

Geometrization of the scalar:

for a generic family of solutions during inflation, the scalar kinetic term can be neglected, so

$$\xi R \phi = -V'(\phi) + \mathcal{O}(|\xi|^{-1}) \ .$$

No conformal transformation, we remain in the the physical (Jordan) frame!

These solutions are the same as for $f(R)$ gravity with

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \;\; f(R) = R - \frac{\xi R\phi^2(R)}{2} - V(\phi(R)).$$

For $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$, this just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$.

The same theorem is valid for a multi-component scalar field.

More generally, $R^2$ inflation (with an arbitrary $n_s, r$) serves as an intermediate dynamical attractor for a large class of scalar-tensor gravity models.

However, this property suddenly breaks immediately after the end of inflation when $R$ becomes negative temporarily during oscillations.

# Inflation in the mixed $R^2$-Higgs model

M. He, A. A. Starobinsky and J. Yokoyama, JCAP **1805**, 064 (2018).

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right) - \frac{\xi R \chi^2}{2} + \frac{1}{2}\chi_{,\mu}\chi^{,\mu} - \frac{\lambda\chi^4}{4}, \ \ \xi < 0, \ \ |\xi| \gg 1$$

Can be conformally transformed to GR with two interacting scalar fields in the Einstein frame. The effective two scalar field potential for the dual model:

$$U = e^{-2\alpha\phi}\left(\frac{\lambda}{4}\chi^4 + \frac{M^2}{2\alpha^2}\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)^2\right)$$

$$\alpha = \sqrt{\frac{16\pi G}{3}}, \quad R = 3M^2\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)$$

# One-field inflation in the attractor regime

In the attractor regime during inflation:

$$\alpha\phi \gg 1, \quad \chi^2 \approx \frac{|\xi|R}{\lambda}, \quad e^{\alpha\phi} \approx \chi^2\left(8\pi G|\xi| + \frac{\lambda}{3|\xi|M^2}\right)$$

that directly follows from the geometrization of the Higgs boson in the physical (Jordan) frame. Thus, we return to the $f(R) = R + \frac{R^2}{6M^2}$ model with the renormalized scalaron mass $M \to \tilde{M}$:

$$\frac{1}{\tilde{M}^2} = \frac{1}{M^2} + \frac{24\pi G\xi^2}{\lambda}$$

Double-field inflation reduces to the single $(R + R^2)$ one for the most of trajectories in the phase space.

# Kinematic origin of scalar perturbations

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different in different points of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then

$$\mathcal{R}(\mathbf{r}) = \delta N_{tot}(\mathbf{r})$$

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B 117, 175 (1982) in the case of one-field inflation.

# Visualizing small differences in the number of e-folds

Duration of inflation in terms of e-folds was finite for all points inside our past light cone. For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta,\phi)}{T_\gamma} = -\frac{1}{5}\mathcal{R}(r_{LSS},\theta,\phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS},\theta,\phi)$$

For $n_s = 1$,

$$\ell(\ell+1)C_{\ell,s} = \frac{2\pi}{25}P_\zeta$$

For $\frac{\Delta T}{T} \sim 10^{-5}$, $\delta N \sim 5 \times 10^{-5}$, and for $H \sim 10^{14}\,\mathrm{GeV}$, $\delta t \sim 5 t_{Pl}$ !

Planck time intervals are seen by the naked eye!

# Different types of quantum corrections to the simplest model

- Logarithmic running of the free model parameter $M$ with curvature.
- Terms with higher derivatives of $R$ considered perturbatively (to avoid the appearance of ghosts).
- Terms arising from the conformal anomaly.

At present, no necessity to break the Lorentz invariance and to introduce additional spatial dimensions at the energy (Hubble) scale of inflation.

# Logarithmic running of $M$ with curvature

Due to the scale-invariance of the $R + R^2$ model for $R \gg M^2$, one may expect logarithmic running of the dimensionless coefficient in front of the $R^2$ term for large energies and curvatures. The concrete 'asymptotically safe' model with

$$f(R) = R + \frac{R^2}{6M^2\left[1 + b\ln\left(\frac{R}{\mu^2}\right)\right]}$$

was recently considered in L.-H. Liu, T. Prokopec, A. A. Starobinsky, Phys. Rev. D **98**, 043505 (2018).

However, comparison with CMB observational data shows that $b$ is small by modulus: $|b| \lesssim 10^{-2}$. Thus, from the observational point of view this model can be simplified to

$$f(R) = R + \frac{R^2}{6M^2}\left[1 - b\ln\left(\frac{R}{\mu^2}\right)\right],$$

for which the analytic solution exists:

$$n_s - 1 = -\frac{4b}{3}\left(e^{\frac{2bN}{3}} - 1\right)^{-1}$$

$$r = \frac{16b^2}{3}\frac{e^{\frac{4bN}{3}}}{\left(e^{\frac{2bN}{3}} - 1\right)^2}$$

For $|b|N \ll 1$, these expressions reduce to those for the $R + R^2$ model.

# Second type: terms with higher derivatives of $R$

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + \alpha R^2 + \gamma R\Box R\right], \quad \alpha = \frac{1}{6M^2}$$

An inflationary regime in this model was first considered in S. Gottlöber, H.-J. Schmidt and A. A. Starobinsky, Class. Quant. Grav. **7**, 803 (1990). But this model, if taken in full, has a scalar ghost in addition to a physical massive scalar and the massless graviton.

Its recent re-consideration avoiding ghosts:
A. R. R. Castellanos, F. Sobreira, I. L. Shapiro and A. A. Starobinsky, JCAP **1812**, 007 (2018).

The idea is to treat the $\gamma R\Box R$ term perturbatively with respect to the $R + R^2$ gravity, i.e., to consider only those solutions which reduce to the solutions of the $R + R^2$ gravity in the limit $\gamma - 0$. Then the second (ghost) scalar degree of freedom does not appear.

Results:

1. $|k| \lesssim 0.3$ where $k = \frac{\gamma}{6\alpha^2}$.

2. In the limit $kN \ll 1$, leading corrections $\propto kN$ to $n_s - 1$ and $r$ vanish. The first result is in the agreement with that in a more general non-local gravity model without ghosts constructed in A. S. Koshelev, L. Modesto, L. Rachwal and A. A. Starobinsky, JHEP **1611**, 067 (2016) which contains an infinite number of $R$ derivatives.

# Third type: terms arising from the conformal (trace) anomaly

The tensor producing the $\propto \left(R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{3}\right)$ term in the trace anomaly:

$$T_\mu^\nu = \frac{k_2}{2880\pi^2}\left(R_\mu^\alpha R_\alpha^\nu - \frac{2}{3}RR_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{4}\delta_\mu^\nu R^2\right)$$

It is covariantly conserved in the isotropic case only! Can be generalized to the weakly anisotropic case by adding a term proportional to the first power of the Weyl tensor (AS, 1981).

$$T_0^0 = \frac{3H^4}{8\pi GH_1^2}, \quad T = -\frac{1}{8\pi GH_1^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{3}\right), \quad H_1^2 = \frac{360\pi}{k_2 G}$$

The spectrum of scalar and tensor perturbations in this case was calculated already in A. A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983).

$$n_s - 1 = -2\beta \, \frac{e^{\beta N}}{e^{\beta N} - 1}, \quad \beta = \frac{M^2}{3H_1^2}$$

If $n_s > 0.957$ and $N = 55$, then $H_1 > 7.2M$.

# Perspectives of future discoveries

- Primordial gravitational waves from inflation: $r$.
  $r \lesssim 8(1 - n_s) \approx 0.3$ (confirmed!) but may be much less. However, under reasonable assumptions one may expect that $r \gtrsim (n_s - 1)^2 \approx 10^{-3}$. The target prediction in the simplest (one-parametric) models is
  $r = 3(n_s - 1)^2 \approx 0.004$.
- A more precise measurement of $n_s - 1 \Longrightarrow$ duration of transition from inflation to the radiation dominated stage $\Longrightarrow$ information on inflaton (scalaron) couplings to known elementary particles at super-high energies $E \lesssim 10^{13}\ \mathrm{Gev}$.
- Local non-smooth features in the scalar power spectrum at cosmological scales (?).
- Local enhancement of the power spectrum at small scales leading to a significant amount of primordial black holes (?).

# Conclusions

- The typical inflationary predictions that $|n_s - 1|$ is small and of the order of $N_H^{-1}$, and that $r$ does not exceed $\sim 8(1 - n_s)$ are confirmed. Typical consequences following without assuming additional small parameters: $H_{55} \sim 10^{14}\,\mathrm{GeV},\ m_{infl} \sim 10^{13}\,\mathrm{GeV}$.
- In $f(R)$ gravity, the simplest $R + R^2$ model is one-parametric and has the preferred values $n_s - 1 = -\frac{2}{N} \approx -0.035$ and $r = 3(n_s - 1)^2 \approx 0.004$. The first value produces the best fit to present observational CMB data. The same prediction follows for the Higgs and the mixed $R^2$-Higgs models though actual values of $N$ are slightly different for these 3 cases.
- Inflation in $f(R)$ gravity represents a dynamical attractor for slow-rolling scalar fields strongly coupled to gravity.

- Comparison with observational data shows that logarithmic high-curvature quantum corrections to the $R + R^2$ model in the observable part of inflation are small, no more than a few percents. The same refers to higher-derivative and conformal anomaly corrections.
- This model does not solve the singularity problem, but in fact this is an advantage for it since it appears that it is much easier to reach the inflationary regime from an anisotropic pre-inflationary space-time which has curvature much exceeding that during inflation.
- Observational predictions for this model have been obtained without assuming the existence of the exact $S$-matrix, that requires the existence of a 'Grand-observer' who knows all information about the initial state and can collect all information about the final state. It is also not assumed that the future time infinity in the Penrose conformal diagram is a point, and not a horizontal line.